## Simulation of Transmission of Daylight through Cylindrical Light Pipes

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Abstract: This paper presents the results of modeling and simulating the transmission of beams of sunlight and diffuse skylight, separately and together, through circular light pipes with and without bends. A well-known theoretical model of the transmission of light rays through straight cylindrical light pipes was introduced by Zastrow and Wittwer in 1986 and re-examined by Swift and Smith in 1995. The present authors propose an alternative approach that differs both in terms of the procedure for the calculation of the transmission of daylight components, and in the manner with which locally measured daylight illuminance data are utilized. The authors apply raytracing principles to trace an individual ray from the entry to the exit of a pipe. Torus sections are used to model bends in cylindrical light pipes. The present method is theoretical but lends itself to practical application and can be used for the design of a particular pipe. The paper illustrates the generation of sky luminance data from locally measured diffuse horizontal daylight though straight pipes and pipes with bends. The computational procedure was coded into a computer program. The program was used to generate some interesting results that include results from the simulation of the transmission of daylight through straight pipes and pipes with bends.

Keywords: daylighting, light pipe, sunlighting, ray tracing, skylighting.

#### 1. Introduction

Daylighting offers great potential for applications in tropical regions because daylight is voluminous and highly available near the equator. Considerable research has been conducted on the luminosity of the tropical sky. However, the usual utilization of diffuse skylight, or light from the sky, for daylighting through a window can provide sufficient illumination only for those areas near the windows. The use of larger windows is not effective and would result in the introduction of excessive heat that contributes to the cooling load of a building. Light pipes and reflector systems that can utilize direct sunlight have been shown to be more effective in bringing daylight into deeper interior spaces [1-4]. However, light pipe application in buildings in tropical regions has not been advocated. In fact, daylighting by whatever means in buildings in cities in tropical Southeast and East Asia countries, such as Singapore, Kuala Lumpur, Hong Kong, and Bangkok has been limited. In such buildings, electric lighting is fully relied on for illuminating the building's interior during the daytime because of the perceived potential contribution to the cooling load due to the penetration of solar radiation.

This paper presents an application of analytical calculus and raytracing principles to the development of models for the transmission of beam sunlight and diffuse skylight, separately and together, through circular mirror light pipes (MLP) with and without bends. Section Two of this paper presents a brief review of the literature for both roof-mounted light guides, and façade-mounted light guides. It utilizes the CIE Report 173:2006 to give an overview of the classification of light pipes and light guides [5]. It then reviews the literature on light guides based on this perspective. Section Three briefly reviews accepted models of skylight luminance and diffuse skylight illuminance on an inclined plane, and describes the basic consideration and the resultant models of light ray transmission that have appeared in the literature. It then presents the development of the proposed methods or models, of calculation of the transmission of light rays. Section Four present the results of the calculation for collimated rays and diffuse rays through straight pipes and pipes with bends. The Section also demonstrates the use of a standard sky luminance model for the calculation of the

luminance of 145 points or patches in the sky dome and the method of the calculation of the transmission of the light rays from each sky patch through the light pipes. Section Five concludes the paper.

### 2. A Brief Review of the Literature

#### 2.1 Daylight availability and models

The availability of daylight globally is discussed in Reas [6]. The term is used to mean the amount of daylight from the sun and daylight from the sky for a specific location, time, and sky condition. Daylight has been reasonably well-studied all over the world and mathematical models on its availability are available. In the tropical regions, its high availability is well-recognized [7-9]. Daylight reaching a surface on the earth is comprised of light from the sun, or direct sunlight, and light from the sky hemisphere, or diffuse skylight. The characteristics of each component have been well studied, [6, 10-16]. Various sky luminance models, including the ASRC-CIE model, have been evaluated against measurement data from North Bangkok [17]. For sky illuminance on an inclined plane, the Perez model with parameters derived from the North Bangkok illuminance data have also been found to perform well [16].

#### 2.2 Light Guides

We turn our attention to hollow light guidance systems that are used to transfer natural daylight from the exterior of a building into the interior. On a broad context, the CIE report examines tubular daylight guidance systems and distinguishes roof-mounted systems from façade-mounted systems [5]. For roof-mounted systems, diffuse skylight from the whole hemisphere as well as direct sunlight can enter the guide, while a simple façade-mounted system receives daylight from the half of the hemisphere in front of it.

We can further classify each of the two systems in accordance to its daylight collection method into those comprising a passive collector and those comprising a collector that optically channels direct sunlight. Daylight that enters a light guide could be transported to a discrete output port or be transported and outputted through the prismatic surface of the guide or through slits along the guide. Light guides include those that comprise lens and mirrors to guide light without a need for physical guide or surfaces. In this paper, we will only consider light pipes that comprise physical pipes or tubes with mirrored surfaces that reflect light.

#### **Light Pipes**

In this review, we will consider mirror light pipes of circular cross-section (or cylindrical shape) and mirror light pipes of a rectangular cross section. Zastrow and Wittwer [18] consider transmissions of light beam across cylindrical light pipes and offer a simple relationship for the transmission of light as a function of the length of the pipe, the diameter of the pipe, and the angle of incidence at the light pipe's entry. Swift and Smith [19] further refine the relationship of Zastrow and Wittwer to more detailed relationships that also account for the pipe's surface, which has spectrally sensitive reflectance. Swift and Smith [20] apply an approach similar to that in their earlier paper to rectangular light pipes and conclude that the illuminance at the exit port can be distinguished into nine sections of different illuminance levels when the rays at the entry port are collimated and uniformly distributed. Light pipes have been individually designed and installed in buildings, but small roof-mounted light pipes comprising passive input domes and discrete output ports are increasingly commercially marketed. Rectangular light pipes fit well horizontally in the rectangular plenum above the ceiling of a building and are thus mostly façade-mounted.

#### **Passive Roof Mounted Light Pipes**

When the configuration of a building permit that is when the number of floors from the roof to the space to be illuminated by daylight is limited, passive roof-mounted light pipes offer an effective means of transferring natural daylight from the roof of a building to spaces at lower levels. These light pipes typically comprise a clear polycarbonate dome at the upper end, a zenithal or vertical circular pipe section, and a diffuser section as exit port for daylight at the lower end. Figure 1(a) illustrates the configuration of a roof-mounted pipe. Such light pipes are commercially available.



(a) Passive roof-mounted light pipe



(b) Plan view of façade-mounted light pipe

Figure 1. Configuration of roof-mounted and façade-mounted light pipes.

#### Façade mounted light pipes

In these light pipes, daylight enters a pipe through openings on the facade. These light pipes find ready application in high latitude regions where the pipes would be oriented towards the equator or the southern direction. The sun would traverse in front of the pipe during the whole year. Such light pipes mostly capture and utilize direct sunlight. Limitations of space result in the use of small apertures at the entry. Reflectors are used to direct sunlight along the axis of the pipe to the interior. Diffuse sections are fitted along the interior sections at the end of a pipe. Figure 1(b) illustrates the configuration of such light pipes. Hien and Chirarattananon [21] utilize raytracing principles coded into a computer program, called BESim, to calculate the transmission of beam sunlight using the measured value of the beam illuminance. For diffuse skylight, the authors use a flux transfer method. The calculated results agree reasonably well with experimental results, where a 5.75 m straight pipe with a square entry port of  $0.5m \times 0.5m$  was used in a series of experiments.

### 3. Models of Skylights and Modeling Daylight Transmission through Mirror Circular Light Pipes

#### 3.1 Models of skylights

Models of skylights that have found general acceptance and have been evaluated against measurement data from a Bangkok location are described below.

### Skylight luminance

The distribution of the luminance of skylights in a sky dome is highly non-uniform. Reference [17] reports an evaluation of 8 sky luminance and radiance distribution models and concludes that no single model is superior to any others under all sky conditions. A set of standard distributions (ISO 15469:2004) has been adopted by the International Organization for Standardization (ISO) and the International Commission on Illumination (CIE), but no appropriate insulation parameter has been associated with this set of distributions [22]. An earlier model called the ASRC-CIE model [23] has been popularly used in energy simulation programs. In this latter model, the luminance of a point in the sky is given as:

$$L(\phi, \gamma, \zeta, \phi_{s}, \gamma_{s}) = b_{cl} L_{cie-cl}(\phi, \gamma, \zeta, \phi_{s}, \gamma_{s}) + b_{ct} L_{cie-ct}(\phi, \gamma, \zeta, \phi_{s}, \gamma_{s}) + b_{in} L_{in}(\phi, \gamma, \zeta, \phi_{s}, \gamma_{s}) + b_{ci} L_{cie-ci}(\phi)$$
(1)

where  $\phi$  is the zenith angle of the given point in the sky,

- $\gamma$  is the azimuth angle of the point,
- $\zeta$  is the angular distance of the point from the sun,
- $\phi_s$  is the solar zenith angle,
- $\gamma_s$  is the solar azimuth angle,

$$L_{cie-cl}(\phi,\gamma,\zeta,\phi_{s},\gamma_{s}), L_{cie-ct}(\phi,\gamma,\zeta,\phi_{s},\gamma_{s})$$

 $L_{in}(\phi, \gamma, \zeta, \phi_s, \gamma_s)$ , and  $L_{cie-ov}(\phi)$  are clear sky, turbid clear sky, intermediate sky, and overcast sky models adopted by CIE, and  $b_{cb}$   $b_{cv}$   $b_{inv}$   $b_{ov}$  are coefficients whose values depend on the values of Perez' clearness index and Perez's brightness index. Perez's indices are derived from the sun and diffuse horizontal sky irradiances and the solar zenith angle.

#### Skylight illuminance on an inclined plane

A commonly accepted model of diffuse skylight illuminance on a plane of given inclination angle  $\beta$ , including the vertical angle, is that of Perez et al. [11]. Perez's model takes the form:

$$E_{vd\beta} = E_{vdh} \left[ \frac{1}{2} \left( 1 + \cos \beta \right) \left( 1 - F_1 \right) + F_1 \left( \frac{a}{c} \right) + F_2 \sin \beta \right]$$
(2)

where  $F_1$ ,  $F_2$ , *a*, and *c* are functions of solar altitude angle, angular position of the sun relative to the normal of the inclined plane, and the values of Perez's clearness index and brightness index. Reference [16] reports that Perez's model performs best when the parameters are derived from the measurement data of Bangkok.

In the use of both models above, measurement data of the sun and diffuse horizontal sky irradiances and illuminances are required. Sunlight is measured directly at a daylight measurement station and this reduces the need for a model. However, in the case that no measurement data are available, there are models for the calculation of both sun and diffuse sky irradiances and illuminances for a clear, an intermediate, or an overcast sky.

# 3.2 Theoretically derived daylight transmission models for straight pipes

Consider a light ray entering a straight circular light pipe in Figure 2.



(a) A model of a cylindrical pipe with a rectangular coordinate



(b) A view of the pipe along coordinate z.

Figure 2. A ray entering a model of the cylindrical pipe.

A light ray enters the entry port of the pipe at position  $\mathbf{p}_0$ . The ray travels in the direction represented by the unit vector  $\mathbf{v}_0$ . The ray reaches point  $\mathbf{p}_1$  on the surface and its reflection reaches the pipe surface at point  $\mathbf{p}_2$ . The length of the projection of vector  $\mathbf{v}_1$  onto the x-y plane is d.

#### **Transmission function of Zastrow and Wittwer**

Zastrow and Wittwer [18] define the average length of projection of the reflection vectors on the x-y plane as  $d_{eff}$ . If the length of the pipe is *l*, and if the incidence angle of the light ray with the axis of the pipe is  $\theta$ , then the approximate number of reflections *n* of the ray from the surface is

$$n \approx \frac{l}{d_{eff} / \tan \theta} = l \tan \theta / d_{eff}$$
(3)

The authors derives  $d_{eff}$  from a consideration of the average length of the projected vector as

$$d_{eff} = \frac{\pi D}{4} \tag{4}$$

The transmission function T is then obtained as

$$T = \rho^{L \tan \theta / d_{eff}} \tag{5}$$

where  $\rho$  is the reflectance of the surface of the pipe.

The transmission function in (5) uses the approximate number of reflections in (3), where the right-hand side is not necessarily an integer value. It was intended as an approximate function for rays that are collimated in one direction. It is also assumed that  $d_{eff}$  pertains to average rays. The reflectance

property of the pipe's surface is assumed to be independent of the angle of incidence and the wavelength of the light rays. The transmission function does not consider the properties of the entry nor the exit ports.

#### Transmission function of Swift and Smith

For Swift and Smith [20], the transmission function in (5) is only an approximation. The authors consider corrections to the use of continuous functions to approximate the integral number of reflections and offer the following relationship for the transmission function:

$$T = \frac{4}{\pi} \int_0^1 \frac{s^2}{\sqrt{d^2 - s^2}} \rho^{\operatorname{int}[p \tan \theta/s]} \{1$$
  
-(1-\rho)(p \tan \theta/s - \int[p \tan \theta/s]\}ds (6)

where p = l/D is the aspect ratio or the ratio of the pipe length to its diameter and where int[x] denotes the integer part of x. This function represents the average transmission for light rays that are collimated in one direction. The integral is not analytically integrable. The authors also demonstrate results for experiments that validate the derived function. Moreover, for pipe surfaces which possess reflectance that is spectrally sensitive at some wavelengths of the light rays, the authors show that the light pipe can cause spectral distortion in the transmitted light due to non-uniform reflections. Kocifaj [24] presents a computer software suit that was developed to calculate the transmission of daylight. The author considers the application of the set of standard sky luminance distributions (SSLD) of ISO15469:2004 to calculate the illuminance due to the skylight at the entry aperture for light rays from each direction of the sky and applies an 'analytical solution' of transmission functions to light rays. Here, the transmission of sunlight is separately calculated. The author suggests the use of a special glazing element at the exit aperture of a light. He proposes that the central area of the element be diffusive, while the peripheral area should be transparent as the transmitted sun rays tend to be concentrated at the central area and transmitted diffuse skylight tends to spread over the exit port. This will enable daylight to be efficiently and diffusively transmitted.

#### 3.3 Application of raytracing to straight pipes

The raytracing method is based on the tracing of the specular reflection of individual rays. This method has been applied to configurations of light pipes that comprise flat surfaces. Dutton and Shao [25] used long thin rectangular sections to form approximate circular-shaped pipes and simulate light pipe transmission by the use of Photopia, a computer program. When the authors simulated the transmission of collimated rays, the results from the Photopia calculation matched very well with those calculated from the transmission function (6) of Swift and Smith. The authors also simulated the transmission of skylight and sunlight, with different solar altitude angles, from the clear sky model through the approximate pipes and compared the results with those calculated from (6). They concluded that both agree well with each other. However, the authors do not elaborate on how the function in (6) is applied with skylight from the clear sky model, since (6) is applicable to rays from a given direction. The authors also compared the simulated results using the clear sky model with a fixed sun angle with the experimental results and found a clear divergence between the two sets of results. The authors attribute this to uncertainties in several parameter values and to the difference between the calculated skylight from assumed sky conditions and actual sky measurements.

#### 3.4 Raytracing for cylindrical light pipes with bends

We apply our raytracing method, also considered to be a theoretical method, to the problem of the transmission of light rays through light pipes with bends. In raytracing, each individual ray is traced along its path of travel, where it is reflected when it encounters a surface. At the point of conjunction with a surface, a part of the electromagnetic radiation in the ray may be absorbed, and another part is specularly reflected. There could be refraction, or diffuse reflection, and other directional reflection behaviors depending on the property of the surface. In general, a sufficiently large number of rays are required to be used in order to capture the transmission characteristics that result from rays that enter from different directions and different pipe configurations and surface properties. The following description illustrates the basis of the method, where we will consider only specular reflection. In our method, a bend connected to a straight cylindrical pipe section is modeled mathematically as a section of torus. This is the surface of revolution generated by revolving a circle in threedimensional space about an axis coplanar with the circle. For the present work, we omit the glazing elements at the entry and exit ports of the pipe.

Consider Figure 2(a) where a rectangular coordinate is located at the base of a cylindrical light pipe of radius r. An imaginary surface covers the entry port of the pipe. A ray with unit directional vector  $\mathbf{v}_0$  enters at point  $\mathbf{p}_0$  on the imaginary surface of the port. The ray reaches the conjunctive point with the interior surface of the pipe at  $\mathbf{p}_1$  and is reflected specularly from the surface with a directional vector  $\mathbf{v}_1$ . The normal vector of the surface at the conjunctive point is denoted  $\mathbf{n}_1$ . Figure 2(b) illustrates the geometrical relationship of the vectors. The conjunctive point  $\mathbf{p}_1$  lies along the line parallel to vector  $\mathbf{v}_0$  and can be obtained from

$$\mathbf{p}_1 = \mathbf{p}_0 + t_o \mathbf{v}_0 \tag{7}$$

where  $t_o$  is a scalar quantity and the point  $\mathbf{p}_1$  lies on the cylindrical surface, so its x and y coordinates follows the governing equation for a cylindrical surface as given in Table 1.

Equation (7) and the surface equation from Table 1 form four equations for the four unknowns of the position vector and  $t_o$ . Once the coordinates of the conjunctive point  $\mathbf{p}_1$  are known, the normal vector of the surface  $\mathbf{n}_1$  at the point can be calculated from the relationship in Table 1. The following equations result from applying the rule of specular reflection:

reflection angle equals incident angle,

 $|\mathbf{v}_1 \cdot \mathbf{n}_1| = \cos \theta = |\mathbf{v}_0 \cdot \mathbf{n}_1|,$ 

the angle between  $\mathbf{v}_1$  and  $\mathbf{v}_0$  is twice the incidence an gle,  $|\mathbf{v}_1 \cdot \mathbf{v}_2| = \cos 2\theta$ , (8)

cross product of  $\mathbf{v}_1$  and  $\mathbf{n}_1$  is normal to  $\mathbf{n}_1$ ,

 $\mathbf{n}_1 \cdot (\mathbf{v}_1 \times \mathbf{n}_1) = 0$ 

The set of equations in (8) can be used to obtain the reflection vector  $v_1$ . When the reflected ray reaches  $p_2$ , the above procedure is used again. This is repeated until the ray reaches the imaginary exit surface.

When the ray reaches the imaginary surface between a cylindrical section and a torus section, the conjunctive point is identified as the point of entry to the torus. Consider Figure 3. For the torus, the distance from the center of the torus to the center of the tube is R, while the radius of the tube that is taken as identical to the radius of the cylinder and to all connected sections of the pipe system is r.



Figure 3. A torus section connected to a cylindrical section.

In Figure 3, ( $x_c$ ,  $y_c$ ,  $z_c$ ) forms the rectangular coordinate of the cylinder. The center of the torus is coincident with the coordinate  $z_{tr}$  that points out from the paper. The center of the tube lies on the plane formed by the  $x_{tr}$  and  $y_{tr}$  coordinates. The coordinate of the torus is rotated from the coordinate of the cylinder by an angle  $\gamma$ . It is also translated with respect to the coordinate of the cylinder. The ray emerging into the torus or from any subsequent section is traced in the same way as is described in the case of cylinder. For each ray, a record of the number of reflections within each section is used to calculate the transmission loss. At the last section of the light pipe, the projection of each ray onto the exit aperture is calculated and summed to eventually form the exitance of light flux from the pipe.

### 4. Calculation and Simulation Results

We first show the results of the calculation of the transmission of collimated light rays, or beam, and random rays through straight pipes and pipes with bends. Then we will illustrate the generation of luminance for points in the sky dome from diffuse horizontal illuminance data, and subsequently, the transmission of daylight through light pipes. For all light pipes in the following example, the values of surface reflectance are all set to 0.95.

# 4.1 Transmission of collimated rays and random rays through straight pipes

#### **Collimated rays**

In order to compare the results from our method of calculation, we first calculate transmission of collimated rays at varying incidence angles with respect to the pipe axis through straight pipes of varying lengths. Figure 4(a) show plots of transmission factors where each line corresponds to transmission factors for collimated rays at the given incidence angles calculated from the function of Zastrow and Wittwer in (5) for pipes of length of 4 times of its radius to that with 40 times. Figure 4(b) show plots of transmission factors from our method based on the use of 50 collimated rays entering the pipe at random positions. The difference between each pair of corresponding points in the graph ranges from 0.0002 to 0.023.

Table 1. Functional description of surfaces and normal vectors for cylindrical and torus surfaces.

Quantity	Cylindrical Section	Torus Section			
Surface function	S: $x^2 + y^2 - r^2$	S: $x^2 + y^2 + z^2 + R^2 - 2R\sqrt{x^2 + y^2} - r^2$			
Surface normal	$\mathbf{n} = -\frac{x}{r}\mathbf{i} - \frac{y}{r}\mathbf{j}$	$\mathbf{n} = \left[ \left(1 - \frac{R}{\sqrt{x^2 + y^2}}\right)^2 (x^2 + y^2) + z^2 \right]^{-\frac{1}{2}} \left[ -x\left(1 - \frac{R}{\sqrt{x^2 + y^2}}\right)\mathbf{i} - y\left(1 - \frac{R}{\sqrt{x^2 + y^2}}\right)\mathbf{j} - z\mathbf{k} \right]^{-\frac{1}{2}} \left[ -x\left(1 - \frac{R}{\sqrt{x^2 + y^2}}\right)\mathbf{i} - y\left(1 - \frac{R}{\sqrt{x^2 + y^2}}\right)\mathbf{j} - z\mathbf{k} \right]^{-\frac{1}{2}} \left[ -x\left(1 - \frac{R}{\sqrt{x^2 + y^2}}\right)\mathbf{i} - y\left(1 - \frac{R}{\sqrt{x^2 + y^2}}\right)\mathbf{j} - z\mathbf{k} \right]^{-\frac{1}{2}} \left[ -x\left(1 - \frac{R}{\sqrt{x^2 + y^2}}\right)\mathbf{i} - y\left(1 - \frac{R}{\sqrt{x^2 + y^2}}\right)\mathbf{j} - z\mathbf{k} \right]^{-\frac{1}{2}} \left[ -x\left(1 - \frac{R}{\sqrt{x^2 + y^2}}\right)\mathbf{j} - z\mathbf{k} \right]^{-\frac{1}{2}} \left[ -x\left(1 - \frac{R}{\sqrt{x^2 + y^2}}\right)\mathbf{j} - z\mathbf{k} \right]^{-\frac{1}{2}} \left[ -x\left(1 - \frac{R}{\sqrt{x^2 + y^2}}\right)\mathbf{j} - z\mathbf{k} \right]^{-\frac{1}{2}} \left[ -x\left(1 - \frac{R}{\sqrt{x^2 + y^2}}\right)\mathbf{j} - z\mathbf{k} \right]^{-\frac{1}{2}} \left[ -x\left(1 - \frac{R}{\sqrt{x^2 + y^2}}\right)\mathbf{j} - z\mathbf{k} \right]^{-\frac{1}{2}} \left[ -x\left(1 - \frac{R}{\sqrt{x^2 + y^2}}\right)\mathbf{j} - z\mathbf{k} \right]^{-\frac{1}{2}} \left[ -x\left(1 - \frac{R}{\sqrt{x^2 + y^2}}\right)\mathbf{j} - z\mathbf{k} \right]^{-\frac{1}{2}} \left[ -x\left(1 - \frac{R}{\sqrt{x^2 + y^2}}\right)\mathbf{j} - z\mathbf{k} \right]^{-\frac{1}{2}} \left[ -x\left(1 - \frac{R}{\sqrt{x^2 + y^2}}\right)\mathbf{j} - z\mathbf{k} \right]^{-\frac{1}{2}} \left[ -x\left(1 - \frac{R}{\sqrt{x^2 + y^2}}\right)\mathbf{j} - z\mathbf{k} \right]^{-\frac{1}{2}} \left[ -x\left(1 - \frac{R}{\sqrt{x^2 + y^2}}\right)\mathbf{j} - z\mathbf{k} \right]^{-\frac{1}{2}} \left[ -x\left(1 - \frac{R}{\sqrt{x^2 + y^2}}\right)\mathbf{j} - z\mathbf{k} \right]^{-\frac{1}{2}} \left[ -x\left(1 - \frac{R}{\sqrt{x^2 + y^2}}\right)\mathbf{j} - z\mathbf{k} \right]^{-\frac{1}{2}} \left[ -x\left(1 - \frac{R}{\sqrt{x^2 + y^2}}\right)\mathbf{j} - z\mathbf{k} \right]^{-\frac{1}{2}} \left[ -x\left(1 - \frac{R}{\sqrt{x^2 + y^2}}\right)\mathbf{j} - z\mathbf{k} \right]^{-\frac{1}{2}} \left[ -x\left(1 - \frac{R}{\sqrt{x^2 + y^2}}\right)\mathbf{j} - z\mathbf{k} \right]^{-\frac{1}{2}} \left[ -x\left(1 - \frac{R}{\sqrt{x^2 + y^2}}\right)\mathbf{j} - z\mathbf{k} \right]^{-\frac{1}{2}} \left[ -x\left(1 - \frac{R}{\sqrt{x^2 + y^2}}\right)\mathbf{j} - z\mathbf{k} \right]^{-\frac{1}{2}} \left[ -x\left(1 - \frac{R}{\sqrt{x^2 + y^2}}\right)\mathbf{j} - z\mathbf{k} \right]^{-\frac{1}{2}} \left[ -x\left(1 - \frac{R}{\sqrt{x^2 + y^2}}\right)\mathbf{j} - z\mathbf{k} \right]^{-\frac{1}{2}} \left[ -x\left(1 - \frac{R}{\sqrt{x^2 + y^2}}\right)\mathbf{j} - z\mathbf{k} \right]^{-\frac{1}{2}} \left[ -x\left(1 - \frac{R}{\sqrt{x^2 + y^2}}\right)\mathbf{j} - z\mathbf{k} \right]^{-\frac{1}{2}} \left[ -x\left(1 - \frac{R}{\sqrt{x^2 + y^2}}\right)\mathbf{j} \right]^{-\frac{1}{2}} \left[ -x\left(1 - \frac{R}{\sqrt{x^2 + y^2}}\right)\mathbf{j} \right]^{-\frac{1}{2}} \left[ -x\left(1 - \frac{R}{\sqrt{x^2 + y^2}}\right]^{-\frac{1}{2}} \left[ -x\left(1 - R$			



(a) Transmission functions calculated from (5)



(b) Transmission functions calculated from raytracing.

**Figure 4.** A comparison of graphs of transmission functions. The numbers on the right of each graph shows the incidence angles.

### **Random Rays**

Figure 5 shows the calculated transmission factors for when 400 random rays are emitted from the Lambertian surface into 200 random locations of entry ports of straight pipes of different lengths. The transmission factor for a pipe of length of 4 times its radius is 0.63. This drops slowly to 0.28 for a pipe with a length of 10 times. At a very rough level, the transmission of random rays is more efficient than that of a beam with incidence angles larger than  $45^{\circ}$ .



Figure 5. Graph of transmission factors for rays from Lambertian surface.

# 4.2 Transmission of collimated rays and random rays through pipes with bends

Figures 6(a), (b), (c), and (d) shows four configurations of light pipes with bends. For Figure 6(a), a straight pipe of length 4r is connected with a  $45^{\circ}$  bend, with a bend radius of 2r. Figure 6(b) shows a pipe with two  $45^{\circ}$  bends and is connected in the middle with a straight pipe of length of 1.67r. Figure 6(c)

is similar to Figure 6(a) but with a 90° bend. Figure 6(d) is also similar to Figure 6(b) but with 90° bends.

#### **Collimated rays**

The transmission factors for collimated rays of different incidence angles are shown in Table 2. Unlike the cases of straight pipes, the transmission factors for pipes with bends do not seem to diminish as much when the incidence angle increases. The pipes in Figure 6(a) and (c) could be compared to a straight pipe with length 4r. However, in these two cases, the rays enter at the same azimuth angles as those of the bends. The values of transmission factors in Table 2 seem to peak at incidence angles of 60° and 45° which reflect the fact that the rays and the exit ports are oriented in the same direction. The values of transmission factors for the pipes in Figure 6(b) and (d) (CF2 and CF4 in Table 2) decrease in comparison to the corresponding values for the pipes in Figure 6(a) and (c) (CF1 and CF3 in Table 2) because of the increasesd length of the pipes and the presence of additional bends, but exhibit similar patterns. For random rays, the bends seem to contribute to decreasing the values of the transmission factors.





(a) A pipe with a 45° bend.

(b) A pipe with two  $45^{\circ}$  bends.



(c) A pipe with a 90° bend.(d) A pipe with two 90° bends.Figure 6. Light pipes with bends.

Table 2. Transmission factors of the pipes in Figure 6.

Ray angle	CF1	CF2	CF3	CF4	
15	0.508	0.447	0.540	0.373	
30	0.520	0.431	0.474	0.385	
45	0.519	0.440	0.553	0.380	
60	0.573	0.452	0.523	0.455	
75	0.572	0.427	0.518	0.404	
Random	0.5547	0.4397	0.5313	0.4130	

# 4.3 Transmission of daylight through straight pipes and pipes with bends

We next consider the transmission of daylight through straight pipes and pipes with bends. From records, the sky of 21 June 2000, the summer solstice, is partly cloudy all day which is typical of the tropical sky of Bangkok, and is thus chosen for illustration.

#### Sky luminance distribution

In order to simulate daylight, we first consider application of the ASRC-CIE model. For a given hour, we first calculate the solar altitude and solar azimuth angles, the Perez's clearness and brightness indices and then calculate the luminance of 145 points in the sky dome. The locations of these 145 points are identical to those in reference [26]. Figure 7 shows a surface plot of the sky luminance distribution at 10.00 of the day where the values of the Perez's indices are 1.5746 and 0.372 respectively and the sky ratio is 0.63. The location is given as: latitude 13.7N, longitude 100.5E, standard longitude 105E. The x and y coordinates are normalized to one.



**Figure 7.** A surface plot of sky luminance distribution for 10.00 hour of 21 June 2000, Bangkok.

### Daylight transmission through straight pipes

In the calculation of entry and transmission of light flux from the sky, 100 rays were used in each of the 145 directions, including that for sunlight. Figure 8(a) shows plots of measured normal sunlight illuminance, horizontal skylight illuminance, calculated skylight and sunlight illuminance through light pipes of lengths 4r and 20r. The values of sky ratio are to be read off from the RHS axis. Both values of sun and sky illuminance reach over 50 klux near noon time. Transmitted values of sky illuminance are approximately 60% for a pipe of length 4r, whereas for a pipe of length 20r this is closer to 40%. Additional calculations were made for 20 March, 27 April, when the sun reaches the zenith position on this day for Bangkok, 15 August (the sun again reaches zenith position), 23 September, and 21 December. The results show that transmission of skylight is more efficient under a cloudy sky than a clear sky, as shown in Table 3. Transmission under an overcast sky is more effective because the zenith luminance of the skylight is higher than those of the other two sky types.

Table 3. Average transmission factors of straight pipes for skylight.

Transmission of	Pipe length					
skylight	4r	8r	12r	16r	20r	
Overall	0.692	0.630	0.582	0.543	0.509	
Overcast sky	0.738	0.682	0.637	0.600	0.567	
Intermediate sky	0.640	0.577	0.529	0.489	0.455	
Clear sky	0.625	0.559	0.508	0.466	0.430	

For sunlight, the transmitted illuminance values appear to be heavily attenuated for the early and late afternoon hours. Near noon, the transmitted values reach 95% for pipe of 4r length and 85% for pipe of 20r length.

#### Daylight transmission through pipes with bends

The bends of the pipes are oriented in the Eastern direction. Figure 8 (b) show plots of calculated values of transmitted

skylight and sunlight through the four light pipes of Figure 6. For the skylight, the transmissions of light pipes with bends seem to be less effective in comparison to straight pipes of comparable length. The larger the number of bends, the lesser is the effectiveness. Transmission of sunlight during the morning and afternoon periods through light pipes with bends oriented towards the Eastern direction is more effective than that for straight pipes. However, the shorter pipes and pipes with a fewer number of bends have transmission efficiencies.



(a) Measured daylight and calculated transmitted daylight



(b) Calculated daylight through pipes with bends.

Figure 8. Measured and calculated daylight though light pipes.

#### 5. Conclusions

This paper gives a brief review of the literature on light pipes and the existing theoretical relationships of transmission of light flux through straight pipes. It proposes an alternative theoretical approach for circular pipes with bends based on raytracing. Even though raytracing is a well-known method that has been applied to configurations comprising flat surfaces of polygonal shapes, we propose a theoretically rigorous application of the raytracing principle to light pipes with bends. We also illustrate a practical application of the method and highlight some salient features of the transmission of collimated rays, random rays, and non-uniformly distributed skylight through light pipes with bends. The authors hope that this paper contributes to existing knowledge and encourage the use of light pipes that will contribute to bringing more daylight into buildings.

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